Interactive Query Formulation using Point to Point Queries

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Abstract

Effective information disclosure in the context of databases with a large conceptual schema is known to be a non-trivial problem. In particular the formulation of ad-hoc queries is a major problem in such contexts. Existing approaches for tackling this problem include graphical query interfaces, query by navigation, and query by construction. In this article we propose the *point to point query mechanism* that can be combined with the existing mechanism into an unprecedented computer supported query formulation mechanism.

In a point to point query a path through the information structure is build. This path can then be used to formulate more complex queries. A point to point query is typically useful when users know some object types which are relevant for their information need, but do not (yet) know how they are related in the conceptual schema. Part of the point to point query mechanism is therefore the selection of the most appropriate path between object types (points) in the conceptual schema.

This article both discusses some of the pragmatic issues involved in the point to point query mechanism, and the theoretical issues involved in finding the relevant paths between selected object types.

1 Introduction

Most present day organisations make use of some automated information system. This usually means that a large body of vital corporate information is stored in these information systems. As a result an obvious, yet crucial, function of information systems is the support of disclosure of this information. Without a set of adequate information disclosure avenues an information system becomes worthless since there is no use in storing information that will never be retrieved. An adequate support for information disclosure, however, is far from a trivial problem. Most query languages do not provide any support for the users in their quest for information. Furthermore, the conceptual schemata of real-life applications tend to be quite large and complicated. As a result, the users may easily become lost in conceptual space' and they will end

up retrieving irrelevant (or even wrong) objects and may miss out on relevant objects. Retrieving irrelevant objects leads to a low *precision*, missing relevant objects has a negative impact on the *recall* ([SM83]).

The disclosure of information stored in an information system has some clear parallels to the disclosure problems encountered in *document retrieval systems*. To draw this parallel in more detail, we quote the information retrieval paradigm as introduced in [BW92]. The paradigm starts with an individual or company having an *information need* they wish to fulfil. This need is typically a vague notion and needs to be made more concrete in terms of an *information request* (the query) in some (formal) language. The information request should be as good as possible a description of the information need. The information request is then passed on to an automated system, or a human intermediary, who will then try to fulfil the information request using the information stored in the system. This is illustrated in the *information disclosure*, or *information retrieval paradigm*, presented in figure 1 which is taken from [BW92].
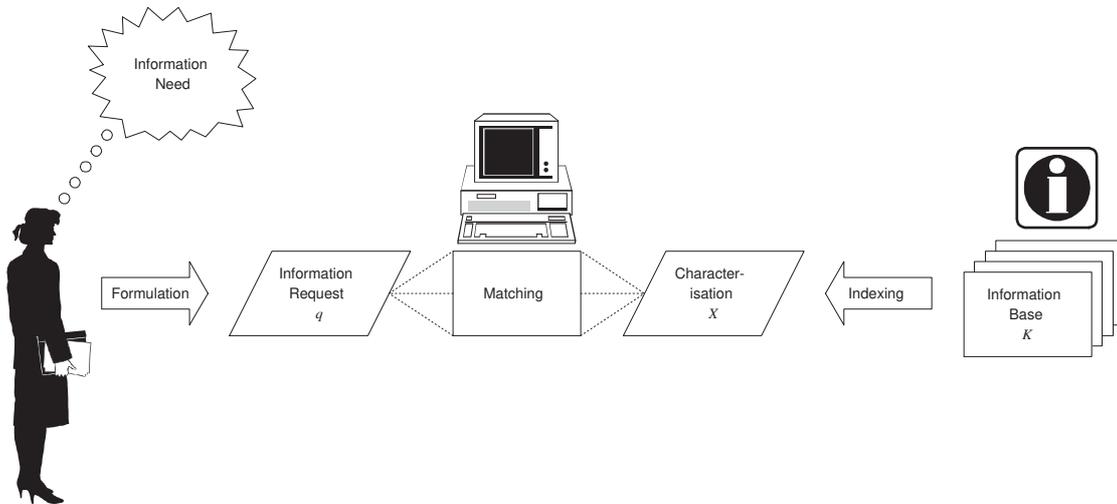

Figure 1: The information retrieval paradigm

We now briefly discuss why the information retrieval paradigm for document retrieval systems is also applicable for information systems. For a more elaborate discussion on the relation between information systems and document (information) retrieval systems in the context of the information retrieval paradigm, refer to [Pro94a]. In the paradigm, the retrievable information is modelled as a set $\mathcal{K}$ of *information objects* constituting the *information base* (or population).

In a document retrieval system the information base will be a set of documents ([SM83]), while in the case of an information system the information base will contain a set of facts conforming to a conceptual schema. Each information object $o \in \mathcal{K}$ is *characterised* by a set of descriptors $\chi(o)$ that facilitates its disclosure. The characterisation of information objects is carried out by a process referred to as indexing. In an information system, the stored objects (the population or information base) can always be identified by a set of (denotable) values, the identification of the object. For example, an address may be identified as a city name, street name, and house number. The characterisation of objects in an information system is directly provided by the reference schemes of the object types.

The actual information disclosure is driven by a process referred to as *matching*. In document retrieval applications this matching process tends to be rather complex. The characterisation of documents is known to be a hard problem ([Mar77], [Cra86]), although newly developed approaches turn out to be quite successful ([Sal89]). In information systems the matching process is less complex as the objects in the information base have a more clear characterisation (the identification). In this case, the identification of the objects (facts) is simply related to the query formulation q by some (formal) query language.

The remaining problem is the query formulation process itself. An easy and intuitive way to formulate queries is absolutely essential for an adequate information disclosure. Quite often, the quest from users to

fulfil their information need can be aptly described by ([Bru93]):

I don't know what I'm looking for, but I'll know when I find it.

In document retrieval systems this problem is attacked by using *query by navigation* ([BW92], [Bru93]) and *relevance feedback* mechanisms ([Rij89]). The query by navigation interaction mechanism between a searcher and the system is well-known from the Information Retrieval field, and has proven to be useful. It shall come as no surprise that these mechanisms also apply to the query formulation problem for information systems. In [BPW93], [BPW94], [HPW94b], [Pro94a] such applications of the *query by navigation* and *relevance feedback* mechanisms have been described before. When combining the query by navigation and manipulation mechanisms with the ideas behind visual interfaces for query formulation as described in e.g. [ADD+92] and [Ros94] powerfull and intuitive tools for computer supported query formulation become feasible. Such tools will also heavily rely on the ideas of direct manipulation interfaces ([Sch83]) as used in present day computer interfaces.

One important step in the improvement of the information disclosure of information systems, is the introduction of query languages on a conceptual level. Examples of such conceptual query languages are RIDL ([Mee82]), LISA-D ([HPW93], [HPW94a]), and FORML ([HHO92]). By letting users formulate queries on a conceptual level, they are safeguarded from having to know the exact mapping to internal representations (e.g. a set of tables conforming to the relational model), as they would be required when formulating queries in a non conceptual language such as SQL. The next step is to introduce ways to support users in the formulation of queries in such conceptual query languages (CQL).

In line with the above discussed information retrieval paradigm and the notion of relevance feedback, a query formulation process (both for a document retrieval system, and an information system) can be said to roughly consist of the following four phases:

1. *The explorative phase.* What information is there, and what does it mean?

2. *The constructive phase.* Using the results of phase 1, the actual query is formulated.

3. *The feedback phase.* The result from the query formulated in phase 2 may not be completely satisfactory. In this case, phases 1 and 2 need to be re-done and the result refined.

4. *The presentation phase.* In most cases, the result of a query needs to be incorporated into a report or some other document. This means that the results must be grouped or aggregated in some form.

Depending on the user's knowledge of the system, the importance of the respective phases may change. For instance, a user who has a good working knowledge of the structure of the stored information may not require an elaborate first phase and would like to proceed with the second phase as soon as possible.

In this paper, we discuss one of the mechanisms to support automated disclosure of information stored in information systems. As stated before, the related notions of *query by navigation* and *query by construction* have already been discussed in [BPW93], [PW95], [Pro94a]. This article is concerned with the *point to point query* (PPQ) mechanism as an additional avenue for information disclosure. A point to point query starts by selecting two or more object types from a conceptual schema. Then the system will return a list of possible (non cyclic) paths through the information structure between the specified object types. For obvious reasons, the paths in this list should be ordered according to some relevance criterion. This style of querying corresponds to a situation in which users know some aspects (object types) about which they want to be informed, but do not yet know the exact details of their information need and the underlying information structure. The query by navigation mechanism, on the other hand, is intended to support users who do not have an overview of the stored information.

Dispite how simple the above scenario may seem, the point to point query mechanism required to realise this query is far from trivial. There are two main problems involved. Firstly, all (non-cyclic) paths through the conceptual schema must be found. This corresponds to finding all (non-cyclic) paths between two

nodes in a graph, which is in general an exponential (NP hard) problem. (Finding the shortest paths is polynomal!) The second main problem is the order in which the results should be presented to the user. It is clear that (especially when there is an abundance of possible paths to choose from) the alternatives should be presented to the user in some order of relevance. We believe to have found an approach to these two problems that makes a point to point query mechanism feasible.

The structure of this article is as follows. In section 2, we discuss an example PPQ session, and elaborate briefly on its integration with query by navigation and query by construction. Section 3 deals with the representation of a conceptual schema as a graph. Searching for a path through this graph is covered in section 4. In section 5 an optimisation strategy is introduced based on a pre-compilation of the conceptual schema graph. Finally, section 6 concludes this article. For the reader who is unfamiliar with the notation style used in this report, it is advisable to first read [Pro94b].

2 An Example PPQ Session

In this section we discuss a sample session involving a point to point query, and also discuss briefly the relationship to query by navigation and query by construction. The discussed example operates on a conceptual schema for the administration of the election of American presidents. The example schema itself is not shown; the structure of the domain will become clear from the sample session. Note that the quality of the verbalisations of paths expressions used in the examples in this section should be improved. However, this is the subject of further research.

In figure 2, a possible screen is depicted for building queries using a point to point query mechanism. The upper window is concerned with the point to point query itself, whereas the lower window contains the complete query under construction. When specifying a point to point query a user specifies a sequence of object types: the points. For each point, the user is offered a listbox containing all object types present in the conceptual schema. The order of the object types in the listbox should preferably be based on some notion of conceptual importance ([CH94]). In figure 3 an existing point to point query path from president to election is extended with another point.

An important underlying practical issue is whether the selection of the points in a point to point query should be done graphically or textually. The theoretical discussions in the remainder of this article are not influenced by such a choice; but this should be taken into serious consideration when implementing the point to point query mechanism. Although the ideal situation may seem to be a graphical selection mechanism using the conceptual schema itself by clicking on object types ellipses, this may turn out to be impossible due to the limited size of PC screens. Graphical based visual query formulation interfaces ([ADD⁺92], [Ros94]) work well for small schemes or queries covering only a small sub schema. However, when a large conceptual schema is involved graphical languages may turn out to be to space consuming.

After all points of the point to point query have been specified, the point to point query can be transformed into a proper query (i.e. a path through the conceptual schema) by pressing the Go! button in the point to point query window. In figure 4, this process is illustrated. The sample PPQ involves three points. Therefore, two paths through the conceptual schema will result. We now shift our attention from the point to point query window to the query by construction window. Note that the small box containing the PPQ abbreviation is now replaced by the paths resulting from the point to point query (i.e. President winning election which resulted in nr of votes). The system initially inserts a most likely path. The user can, however, select alternative paths using a listbox. Note that not all alternative paths between the two points are listed in the listbox. The reason for this is the NP completeness of the path searching problem. To avoid the NP completeness problem, only the best paths are listed initially. However, potentially all paths can be selected (which still remains NP complete) by repeatedly selecting the MORE option. In the remainder of this article we will discuss this in more detail.

Since every path resulting from a query by navigation session connects two points in the conceptual schema, any path through the conceptual schema displayed in the query by construction screen can be used as a
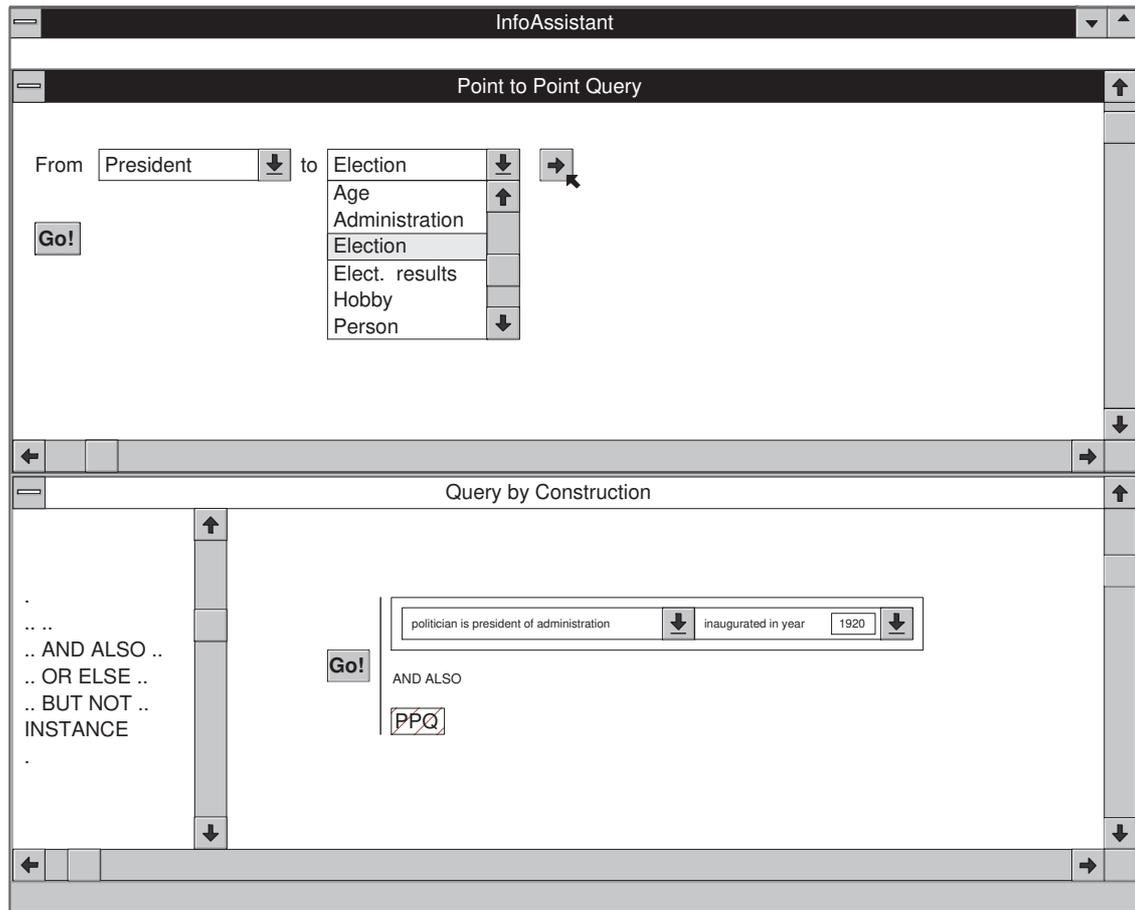
Figure 2: Building a PPQ query

starting point for a query by navigation session, and vice versa. This is illustrated in figure 5. In this session, the user has selected the box which contains the two paths politician is president of administration and inaugurated in year for a query by navigation session. The upper window now displays a node in the query by navigation session, with the path politician is president of administration inaugurated in year as its focus. If the user had selected the inaugurated in year listbox, the initial focus would have been administration inaugurated in year.

The query by construction window is basically a syntax directed editor. In the left part of the window all possible constructs from the query language are listed. In our examples we have used the constructs defined in LISA-D. Once the FORML and LISA-D languages have been merged, a more complete language for the query by construction part will result.

3 A Conceptual Schema as a Graph

For the purpose of finding a path between object types in a conceptual schema, the schema first needs to be translated to a graph. We start out from a formalisation of ORM based on the one used in ([HP95]). However, since only a very limited part of the formalisation is needed, we do not cover the formalisation in full detail.

A conceptual schema is presumed to consist of a set of types $\mathcal{TP}$. Within this set of types two subsets can be distinguished: the relationship types $\mathcal{RL}$, and the object types $\mathcal{OB}$. Furthermore, let $\mathcal{RO}$ be the
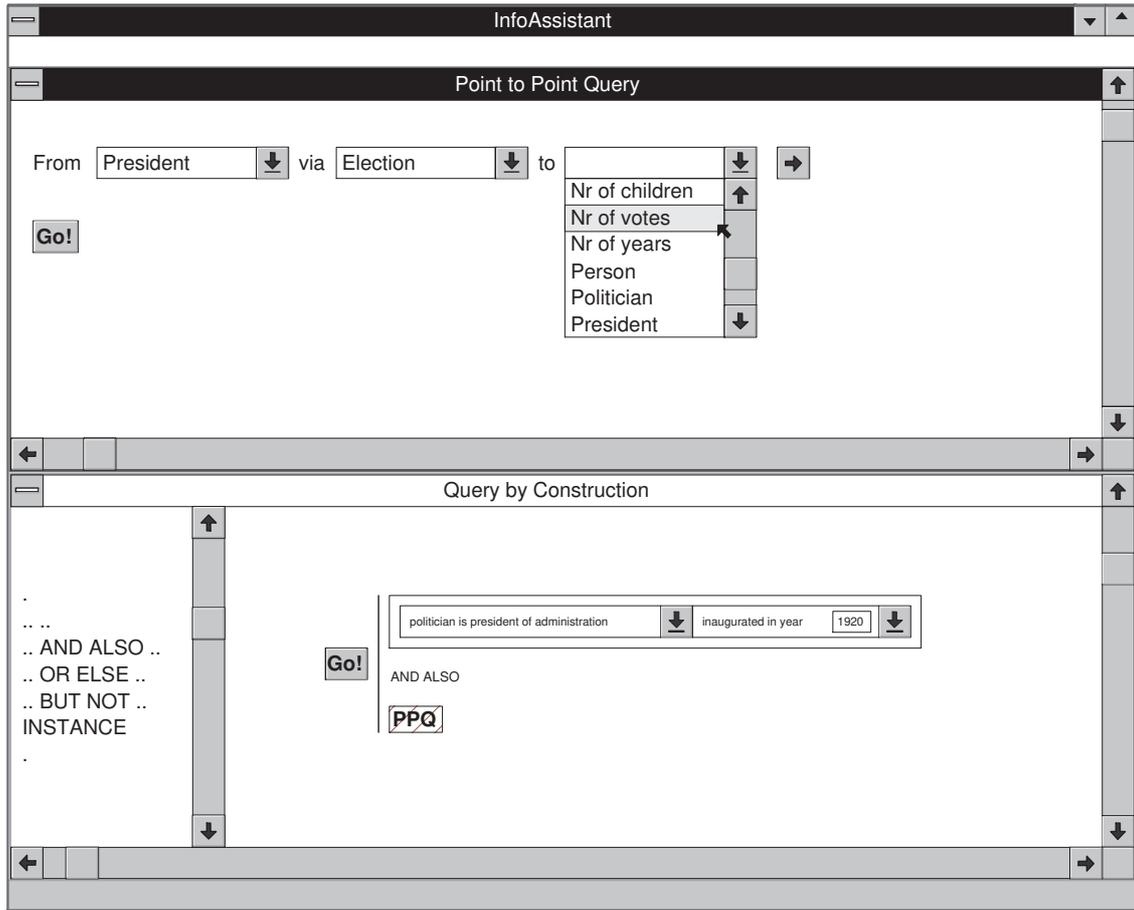
Figure 3: Extending the PPQ path

set of roles in the conceptual schema. The fabric of the conceptual schema is then captured by two functions and two predicates. The set of roles associated to a relationship type is provided by the partition: $\mathsf{Roles} : \mathcal{RL} \to \wp(\mathcal{RO})$. Using this partition, we can define the function Rel which returns for each role the relationship type in which it is involved: $\mathsf{Rel}(r) = f \iff r \in \mathsf{Roles}(f)$. Every role has an object type at its base called the player of the role, which is provided by the function: $\mathsf{Player} : \mathcal{RO} \to \mathcal{TP}$. Subtyping and polymorphism of object types is captured by the predicates $\mathsf{SpecOf} \subseteq \mathcal{OB} \times \mathcal{OB}$ and $\mathsf{HasMorph} \subseteq \mathcal{OB} \times \mathcal{OB}$ respectively. For any ORM conceptual schema the following (undirected) labelled graph $G = \langle N, E \rangle$ can be defined:

$$N \triangleq \mathcal{TP} \tag{1}$$

$$E \triangleq \left\{ \langle \{\mathsf{Player}(r), \mathsf{Rel}(r)\}, r \rangle \mid r \in \mathcal{RO} \right\} \tag{2}$$

$$\cup \quad \left\{ \langle \{x, y\}, \mathsf{SpecOf} \rangle \mid x \,\mathsf{SpecOf}\, y \right\} \tag{3}$$

$$\cup \quad \left\{ \langle \{x, y\}, \mathsf{HasMorph} \rangle \mid x \,\mathsf{HasMorph}\, y \right\} \tag{4}$$

The edges in the resulting graph have the format $\langle \{x, y\}, l \rangle$, where x and y are the source/destination (no order) of the edge, and l is the label of the edge. The labels on the edges either result from the roles in the relationship types (2), or they result from specialisation and polymorphism (3,4). In the remainder, the graph G will be used as an implicit parameter for all introduced functions and operations. As a convention, the nodes of graph G are accessed by $G.N$, and the edges by $G.E$.
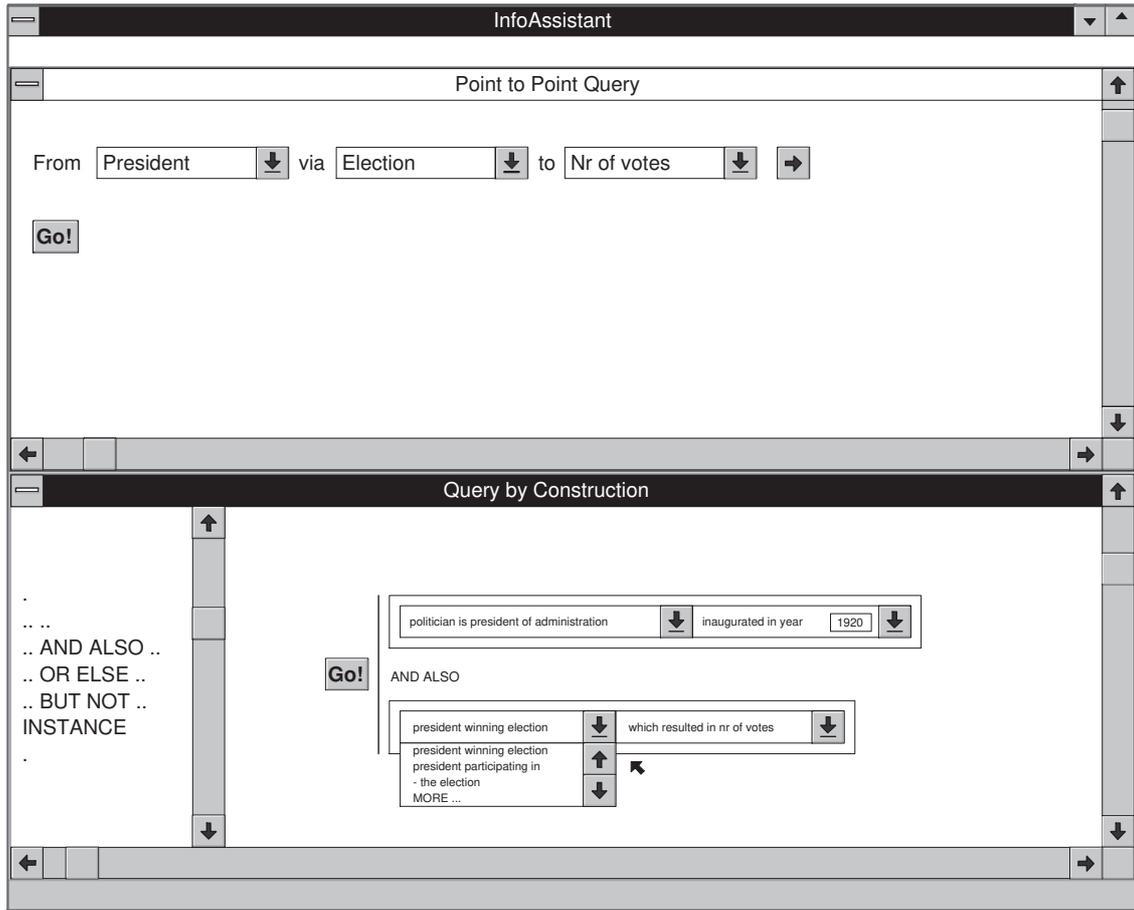
Figure 4: Completing a PPQ

As an example consider the conceptual schema depicted in figure 6. For this schema we have:

$$
\begin{aligned}
\mathcal{TP} &= \{A, B, C, D, f, g\} & \mathsf{Roles}(f) = \{r, s\}, \mathsf{Roles}(g) = \{t, u\} \\
\mathcal{RL} &= \{f, g\} & \mathsf{Player}(r) = A, \mathsf{Player}(s) = B, \mathsf{Player}(t) = C, \mathsf{Player}(u) = A \\
\mathcal{OB} &= \{A, B, C, D\} & A \mathsf{\ HasMorph\ } C, A \mathsf{\ HasMorph\ } g \\
\mathcal{RO} &= \{r, s, t, u\} & D \mathsf{\ SpecOf\ } B
\end{aligned}
$$

From this schema the graph as depicted in figure 7 can be derived.

For point to point queries paths in the graph need to be found. In this article, a path through the graph is denoted as a sequence of alternating nodes and labels:

$$[x_0, l_1, x_1, \ldots, l_n, x_n]$$

Note that if x is a node, then $[x]$ denotes the path consisting of node x only. In the remainder $+\!\!+$ is used as the concatenation operation for sequences. Not all alternating sequences of nodes and labels correspond to a proper path. For a path to be a proper one, it must adhere to two properties:

1. The nodes in the path must originate from the graph: $\forall_{0 \leq i \leq n} [\, x_i \in G.E \,]$.

2. The labels in the path must originate from the proper edges in the graph:

$$\forall_{1 \leq i \leq n} [\, \langle \{x_{i-1}, x_i\}, l_i \rangle \in G.N \,]$$
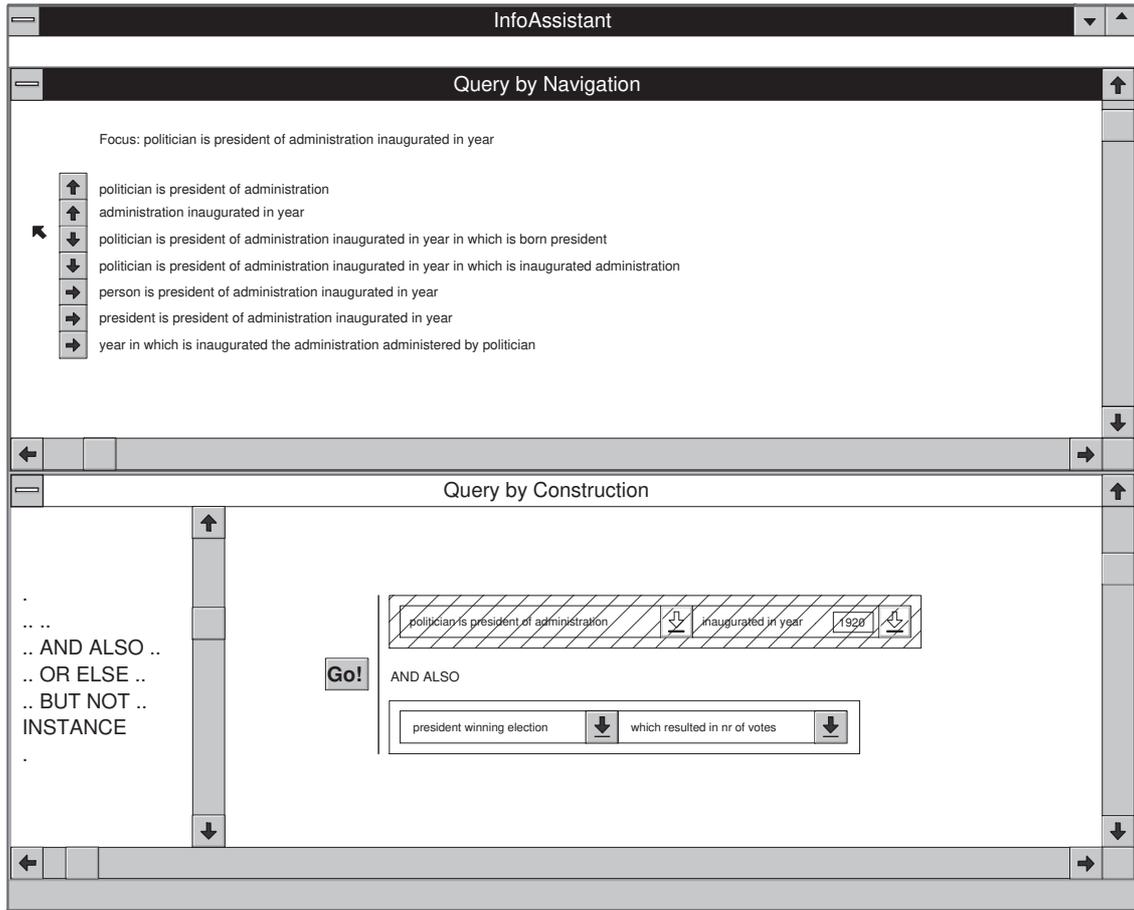
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Figure 5: Switching to query by navigation

In the remainder of this article, Path denotes the set of all valid paths for any graph G resulting from an ORM schema. On such paths the following three functions can be defined: $\text{Begin} : \text{Path} \to G.N$, $\text{End} : \text{Path} \to G.N$, and $\text{Length} : \text{Path} \to \mathbb{N}$, which are identified by:

$$\text{Begin}([x_0, l_1, x_1, \ldots, l_n, x_n]) \triangleq x_0$$

$$\text{End}([x_0, l_1, x_1, \ldots, l_n, x_n]) \triangleq x_n$$

$$\text{Length}([x_0, l_1, x_1, \ldots, l_n, x_n]) \triangleq n$$

Furthermore, the $\in$ and $\notin$ operations can be extended to paths as well, expressing the occurrence of a node on a path:

$$x \in [x_0, l_1, x_1, \ldots, l_n, x_n] \iff x \in \{x_0, \ldots, x_n\}$$

$$x \notin p \iff \neg(x \in p)$$

A *badness* level is associated with every path through the conceptual schema, expressing its conceptual irrelevance. The badness is used to order the alternative paths in the listboxes. Badness is defined in terms of a penalty point system where a high penalty point score corresponds to a low conceptual relevance. Two ways of earning penalty points exist: the relative conceptual irrelevance of the object types in the path, and the length of the path.

For the first class of penalty points the existence of a function: $\text{CWeight} : \mathcal{TP} \to \mathbb{N}$ is presumed. This function should capture the conceptual importance of the types in the conceptual schema, which can for
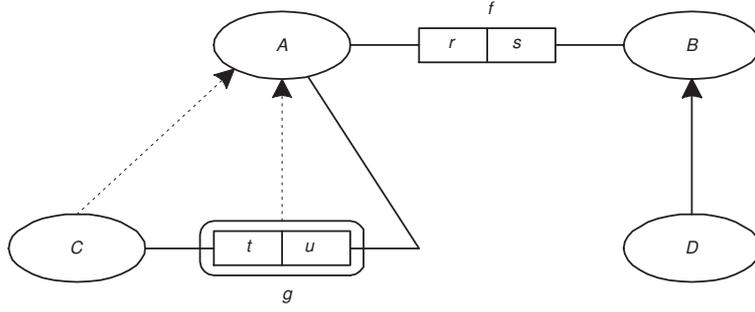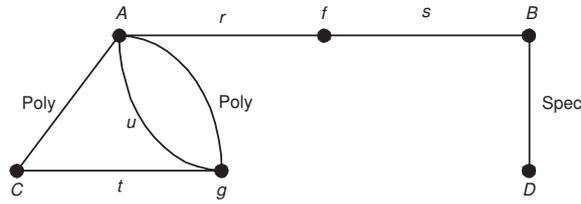
Figure 6: Example Conceptual Schema

Figure 7: Example Graph

instance be derived from the abstraction level at which the type is present ([CH94]). For each object type occurring in a path, the number of penalty points added to the total badness of the path depends on the deviation of its conceptual importance from the maximum conceptual importance in the conceptual schema.

The second way for a path to earn penalty points is the length of the path. For every object type occurring in the path a basic amount of penalty points is added. In order to maintain uniformity of the penalty points added, this basic amount is set equal to the maximum conceptual importance of object types in the schema. Finally, sometimes one would like to be able to control the influence of the two ways to earn penalty points in the final outcome. For this purpose, we introduce the (user definable) constant $C_{weight} \in [0, 1]$. This leads to the following definition of the badness of a non-cyclic path in a graph G:

$$\mathsf{Badness} : \mathsf{Path} \to \mathbb{N}$$

$$\mathsf{Badness}(p) \triangleq \Sigma_{o \in p} \left(C_{weight} \times (MaxCWeight - \mathsf{CWeight}(o)) + (1 - C_{weight}) \times MaxCWeight \right)$$

where $MaxCWeight = \max_{x \in G.N} \mathsf{CWeight}(x)$. Note the $\in$ operation used in the expression $o \in p$ is the above defined inclusion operation for paths through graphs, and that an object type o only occurs at most once in p and that therefore the summation over $o \in p$ is correct with respect to the length of the path. An important property of the $\mathsf{Badness}$ function is the following:

Lemma 3.1 The function $\mathsf{Badness}$ is monotonous strict increasing, i.e.:

if $p +\!\!+ q \in \mathsf{Path}$ is an acyclic path and q is non-empty, then $\mathsf{Badness}(p) < \mathsf{Badness}(p +\!\!+ q)$

Proof:
Follows directly from the definition of $\mathsf{Badness}$ and the observation that the set $\left\{ o \mid o \in p \right\}$ is a proper subset of $\left\{ o \mid o \in p +\!\!+ q \right\}$ □

The above property allows us to incrementally search for the paths with the lowest badness since the badness of a path never decreases when extending it. Note that one might also want to introduce additional fitness factors. For instance, one could take the correlation of the (verbalisation of the) path to a set of keywords describing the users interests into consideration. However, the badness should remain a strict monotonous increasing function.

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4   The Quest

This section is concerned with finding a path through the conceptual schema (graph) between two points (nodes). Although a point to point query typically involves more then two object types, it can always be expressed as a combination of a set of point to point queries over two points. As an example consider figure 4. The newly added point to point query involves three points and is represented as two point to point queries (listboxes) over two points.

In searching paths between two points (nodes) in the graph, an incremental strategy is followed. Two pools of paths are maintained during the entire search: a pool P of paths which could lead to a possible solution and a pool S of found solutions. In every step (increment) of the algorithm these pools are updated. The best (lowest badness) potential solutions in pool P are selected for further extension. By selecting the best paths in P for further extension it can be guaranteed that the first solutions found are the ones with the lowest badness. Within a pool of possible solutions P, and in the context of a graph G, the set of best candidates are provided by:

$$\mathsf{Best} : \wp(\mathsf{Path}) \to \wp(\mathsf{Path})$$
$$\mathsf{Best}(P) \triangleq \left\{ p \in P \;\middle|\; \mathsf{Badness}(p) = \min_{q \in P} \mathsf{Badness}(q) \right\}$$

The first operation we introduce calculates the increment as described above for a pair of pools. It tries to extend the paths in P, and updates the set of found solutions in S if new ones have been found. For any start node f and end node t, we define the increment operator as:

$$\mathsf{Increment}_{f,t:} : \wp(\mathsf{Path}) \times \wp(\mathsf{Path}) \to \wp(\mathsf{Path}) \times \wp(\mathsf{Path}) \times \wp(\mathsf{Path})$$
$$\mathsf{Increment}_{f,t}(P,S) \triangleq \begin{cases} \langle P', S', R' \rangle & \text{if } P \neq \varnothing \\ \langle P, S, S \rangle & \text{otherwise} \end{cases}$$

where:

$$
\begin{aligned}
N &= \left\{ p \,+\!+\, [n, l] \;\middle|\; p \in \mathsf{Best}(P) \wedge \langle \{\mathsf{End}(p), n\}, l \rangle \in G.E \wedge n \notin p \right\} \\
S' &= S \cup \left\{ s \in N \;\middle|\; \mathsf{End}(s) = t \right\} \\
P' &= (P \cup N) - \mathsf{Best}(P) - S' \\
R' &= \left\{ r \in S' \;\middle|\; \mathsf{Badness}(r) \leq \min_{q \in P'} \mathsf{Badness}(q) \right\}
\end{aligned}
$$

For defining the set of (best) extended paths N, all best paths ($p \in \mathsf{Best}(P)$) in the existing pool of possible solutions are extended with an appropriate edge from the graph ($\langle \{\mathsf{End}(p), n\}, l \rangle \in G.E$) while maintaining acyclicity ($n \notin p$). The new set of solutions (S') is simply the old set of solutions extended with the solutions found after extending the best paths. In the new pool of possible solutions (P') the newly found solutions are removed since they should not be extended any further. Although a path in S' has the proper begin and end point it is not considered to be a proper solution until it has a lower badness then the paths in the pool of potential solutions P'. The set of proper solutions is returned in R'.

For the increment operation we have the following property:

Lemma 4.1   If $\mathsf{Increment}_{f,t}(P, S) = \langle P', S', R' \rangle$ and $\mathsf{Increment}_{f,t}(P', S') = \langle P'', S'', R'' \rangle$, then:

$$R' \subseteq R'' \wedge \forall_{r \in R'' - R'} \left[\mathsf{Badness}(r) > \max_{q \in R'} \mathsf{Badness}(q) \right]$$

Proof:

We first prove that $R' \subseteq R''$.

If $r \in R'$, then since $R' \subseteq S' \subseteq S''$ we also have $r \in S''$. Furthermore, from the definition of R' follows: $\mathsf{Badness}(r) \leq \min_{q \in P'} \mathsf{Badness}(q)$. Due to the monotonic behaviour of the $\mathsf{Badness}$ function, we immediately have: $\mathsf{Badness}(r) \leq \min_{q \in P''} \mathsf{Badness}(q)$, since P'' contains the extended paths. From the definition of R' then follows that $r \in R'$.

Now we prove that $\forall_{r \in R'' - R'} \left[\mathsf{Badness}(r) > \max_{q \in R'} \mathsf{Badness}(q) \right]$. If $r \in R'' - R'$, then $r \notin R'$. From this and the definition of R' follows that $r \notin S'$ or $\mathsf{Badness}(r) > \min_{q \in P'} \mathsf{Badness}(q)$. So we have:

1. Let $r \notin S'$. Since $r \in R'' - R'$, we know that r is a newly found solution in R''. So there is a $p \in \mathsf{Best}(P')$ and an $e \in G.E$ such that $p +\!\!+ [e] = r$. From the monotonicity of $\mathsf{Badness}$, it then follows $\mathsf{Badness}(p) < \mathsf{Badness}(r)$.

   If $x \in R'$, then from the definition of R' follows that $\mathsf{Badness}(x) \leq \min_{q \in P'} \mathsf{Badness}(q)$. Since we just concluded that $\mathsf{Badness}(p) < \mathsf{Badness}(r)$ for a certain $p \in P'$, we at least have $\min_{q \in P'} \mathsf{Badness}(q) < \mathsf{Badness}(r)$ Since we also have $\mathsf{Badness}(x) < \mathsf{Badness}(r)$, we in particular have: $\mathsf{Badness}(r) > \max_{q \in R'} \mathsf{Badness}(q)$.

2. Let $\mathsf{Badness}(r) > \min_{q \in P} \mathsf{Badness}(q)$. From the definition of R' follows that if $x \in R'$ then $\mathsf{Badness}(x) \leq \min_{q \in P'} \mathsf{Badness}(q)$, which means that $\mathsf{Badness}(x) < \mathsf{Badness}(r)$. From this finally follows: $\mathsf{Badness}(r) > \max_{q \in R'} \mathsf{Badness}(q)$.

$\square$

This property implies that the result (the R) of a point to point query is returned in monotonous increasing steps. Which means that when presenting the results to the user, the list box can be filled in incremental steps by repeatedly selecting the MORE option.

The increment operation, as such, can not yet be used to calculate the best solutions which are presented in the listboxes as depicted in figure 4. For this latter purpose the $\mathsf{List}_{f,t}(P, S)$ operation is introduced, which serves as a 'driver' function for the entire process.

$$\mathsf{List}_{f,t} : \wp(\mathsf{Path}) \times \wp(\mathsf{Path}) \times \wp(\mathsf{Path}) \to \wp(\mathsf{Path}) \times \wp(\mathsf{Path})$$

$$\mathsf{List}_{f,t}(P, S, R) \triangleq \begin{cases} \langle P', S', R' \rangle & \text{if } R' \neq R \\ \mathsf{List}_{f,t}(P', S', R') & \text{else if } P \neq \varnothing \\ \bot & \text{otherwise} \end{cases}$$

where $\langle P', S', R' \rangle = \mathsf{Increment}_{f,t}(P, S)$

This function calls the increment operation until it has come up with some new solutions ($R' \neq R$) or the pool of potential solutions has been exhausted ($P = \varnothing$). To provide the initial filling for the listboxes of a point to point query from f to t, this function should be applied as:

$$\langle P, S, R \rangle = \mathsf{List}_{f,t}(\{[f]\}, \varnothing, \varnothing)$$

Now R contains the set of found paths to be listed in the listbox. If users desire to see more options they can select the MORE option (see figure 4). This results in another call of the $\mathsf{List}_{f,t}$ function using P, S, R as parameters.

Finally, the paths resulting from the search through the graph need to be translated into linear path expressions. For more details on linear path expressions please refer to [HPW93]. In a later stage, however, the current definition of the linear path expressions as provided in [HPW93] needs to be changed to better match our requirements. Every (proper) path through the graph can be translated into a linear path expression by the following recursive function:

$$\mathsf{PathExpr} : \mathsf{Path} \to \mathsf{PathExpressions}$$

$$\mathsf{PathExpr}([x_0, l_1, x_1, \ldots, l_n, x_n]) \triangleq x_0 \; \mathsf{Connector}(l_1, x_1) \; x_1 \; \ldots \; \mathsf{Connector}(l_n, x_n)$$

where

$$\mathsf{Connector}(l, x) \triangleq \begin{cases} \circ & \text{if } l \in \{\mathsf{HasMorph}, \mathsf{SpecOf}\} \\ \circ\, l\, \circ & \text{if } x \in \mathcal{RL} \wedge l \in \mathsf{Roles}(x) \\ \circ\, l^\leftarrow \circ & \text{otherwise} \end{cases}$$

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Note that when $n = 0$, we have: $\mathsf{PathExpr}([x_0]) = x_0$.

The linear path expressions are for internal use only. They can be mapped to proper SQL queries on the one hand, and verbalised as semi-natural language sentences using the verbalisation information as provided in the conceptual schema on the other hand. As stated before, the verbalisation of path expressions is the subject of further research.

5 Optimisation by Pre-Compilation

When humans look at a conceptual schema to find a path between two object types, they are usually able to identify parts of the schema which can safely be ignored when a searching for the actual path. In figure 8 such a situation is illustrated. In this schema, F is the starting point of the point to point query and T the end point. The three clouds represent subschemes. It is clear that subschema III can be safely ignored when searching for a path from F to T since the only way in/out of subschema III is through object type B. If a path would enter subschema III through B (either continuing via fact type f or g), the path would not be able to leave the subschema without creating a cycle. Such situations are not rare for real life applications. For instance, [Hal95] contains quite a number of schemes of real life applications with a similar pattern.

In this section a strategy is developed that allows us to reduce the graph associated with a conceptual schema before actually commencing a search. A possible way to approach this is to define a pruning algorithm that repeatedly cuts of irrelevant leaves from the graph. However, in the situation sketched in figure 8, subschema III contains a cycle making it impossible for such a simple pruning algorithm to remove the entire subschema III. In this section we therefore develop a strategy for the removal of parts of the graph that may contain cycles. First a clustering algorithm is developed. After this, we use the simple pruning algorithm to remove irrelevant leaves, resulting in the removal of irrelevant subschemas even when they contain cycles (for instance subschema III).

5.1 Clustering a Graph

The first step in our approach is the clustering itself. Clustering can be done by a pre-compilation of the conceptual schema. This pre-compilation should be done after the conceptual schema has been finished, but before the users start formulating queries. Although a conceptual schema is expected to evolve in the course of time ([FOP92], [Pro94a], [OPF94]), the pre-compilation we propose here will not be costly to do after each evolution step. For small schemes one might even consider combining the pre-compilation with the search though the conceptual schema itself.

Formally, a clustering of a graph can be modelled as a function: $C : \mathbb{N} \rightarrowtail \wp(\mathsf{Node})$. An existing cluster i within an existing clustering C can be extended with nodes E, using the $C \bigoplus_i E$ operation. This operation is identified by:

$$(C \oplus_i E)(j) \triangleq \mathsf{if}\ i = j\ \mathsf{then}\ C(j) \cup E\ \mathsf{else}\ C(j)\ \mathsf{fi}\quad \mathsf{for\ each}\ j \in \mathbb{N}$$

In the returned clustering the existing cluster $C(i)$ is grown to $C(i) \cup E$.

A clustered graph can in itself be seen as a graph. The nodes are the clusters (effectively subgraphs of the original graph), and the edges can be derived from the original graph by having an edge between nodes (clusters) in the hypergraph if nodes in the clusters are connected in the original graph. This corresponds to the notion of a hypergraph since the clusters are treated as nodes. Obviously, the hypergraph can also be clustered, leading to yet another hypergraph. In the algorithms discussed here, we will repeatedly make use of the hypergraph notion. The idea is to use the hypergraphs to repeatedly simplify the graph, allowing us to identify irrelevant subschemas (which will correspond to nodes in one of the hypergraphs).

An important concept when clustering is the degree of a node. Let $G.E$ be the set of edges in a graph, then we define the degree of a node n within the context of a set of nodes N to be the number of nodes in N that
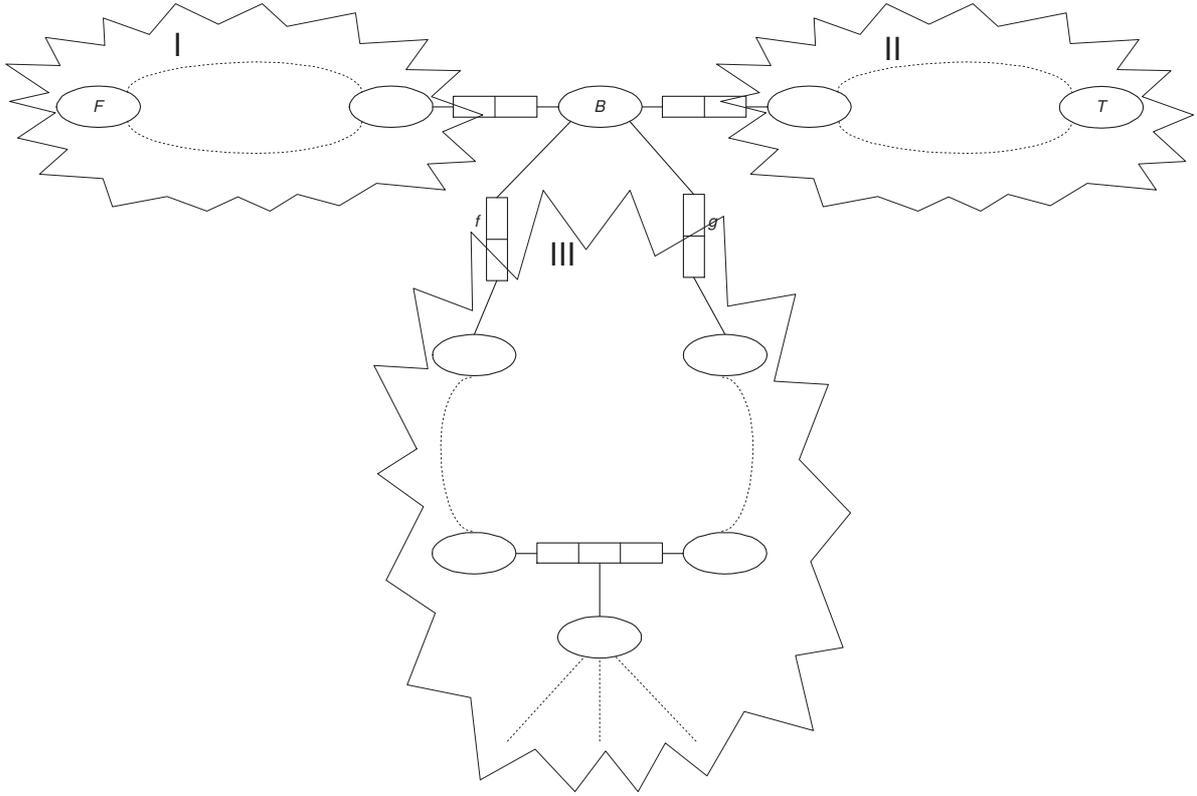
Figure 8: Connected subschemas

can be reached from n by an edge in $G.E$. The word *reached* should be interpreted here as either directly connected, or one of the nodes contained in an involved cluster is connected. This notion of degree can be captured formally as:

$$\mathsf{Deg} : \wp(\mathsf{Node}) \times \mathsf{Node} \to \mathbb{N}$$
$$\mathsf{Deg}(N, n) \triangleq \big| \big\{ m \in N - \{n\} \;\big|\; m \leftrightsquigarrow n \big\} \big|$$

The principle of nodes (which could actually be clusters) being reachable from other nodes is represented by the $\leftrightsquigarrow$ operator:

$$n \leftrightsquigarrow m \iff \exists_{x,y} \big[\, \{x, y\} \in \pi_1(G.E) \wedge x \prec n \wedge y \prec m \,\big]$$

The expression $x \prec y$ captures the intuition of a node x being equal to node y or node x being contained in the cluster y (note that we will later on use hyper clustering, resulting in nodes which contain other nodes). The $\prec$ operator is therefore defined by the following recursive rule:

$$n \prec m \iff n = m \vee \exists_{m' \in m} \big[n \in m' \big]$$

From the above defined notion of degree we derive the so called *normalised* degree of a node. The idea behind the normalised degree is that the leaves of a graph (nodes with $\mathsf{Deg}(N, n) = 1$) can safely be ignored by the clustering algorithm as they will never lead to a cycle. For any node with a higher degree, a closer investigation is required, i.e. the clustering algorithm needs to be applied. Informally, the normalised degree is the number of neighbouring nodes with a degree higher then 1. The formal definition of the normalised degree is:

$$\mathsf{NDeg} : \wp(\mathsf{Node}) \times \mathsf{Node} \to \mathbb{N}$$
$$\mathsf{NDeg}(N, n) \triangleq \big| \big\{ m \in N - \{n\} \;\big|\; \mathsf{Deg}(N, m) > 1 \wedge m \leftrightsquigarrow n \big\} \big|$$
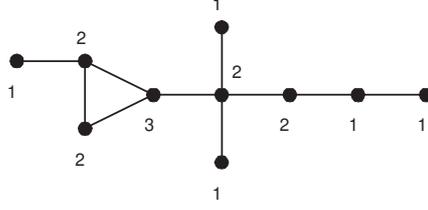
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Figure 9: Normalised degrees of nodes.

As an example, consider figure 9. There a graph is depicted where each node is labelled with the NDeg of the node.

We now have enough primitives to define the clustering algorithm itself. The clustering needs to be done in such a way that the clusters themselves contain no branches (nodes n with $\mathsf{NDeg}(n) > 2$). For clustering three functions are introduced. The first function is simply used to provide a nice interface to the other two clustering functions:

$$\mathsf{Cluster} : \wp(\mathsf{Node}) \to (\mathbb{N} \rightarrowtail \wp(\mathsf{Node}))$$

$$\mathsf{Cluster}(N) \triangleq \mathsf{DoCluster}(\langle N, C \rangle, 1)$$

where C is the initial clustering defined as $C(i) = \varnothing$ for $1 \leq i \leq |N|$.

To cluster a graph G, this function should be invoked as: $\mathsf{Cluster}(G.N)$. The second clustering function (DoCluster) is the 'driver' function of the clustering algorithm, and the third function (Propagate) the 'propagation' function. The driver function takes three parameters. The first parameter represents the set of nodes that have not been placed in any cluster yet. The second parameter is the current clustering, and the last parameter is the number of the cluster that is currently being formed. This function selects, if there is any node left to be clustered, a node with the minimal degree and forms a cluster for this node (using the specified cluster number). Each time a new cluster is formed, the 'propagation' function Propagate is called, which will then try to extend the newly formed cluster. The driver function is identified by:

$$\mathsf{DoCluster} : (\wp(\mathsf{Node}) \times (\mathbb{N} \rightarrowtail \wp(\mathsf{Node}))) \times \mathbb{N} \to (\mathbb{N} \rightarrowtail \wp(\mathsf{Node}))$$

$$\mathsf{DoCluster}(\langle N, C \rangle, i) \triangleq \begin{cases} C & \text{if } N = \varnothing \\ \mathsf{DoCluster}(\mathsf{Propagate}(N - \{n\}, C \oplus_i \{n\}, i), i+1) & \text{otherwise} \end{cases}$$

where $n \in N$ such that $\mathsf{NDeg}(N, n) \leq \min_{m \in N} \mathsf{NDeg}(N, m)$.

The propagation function tries to extend the size of the cluster. It does so by trying to find unclustered nodes which have a normalised degree that is less then or equal two, and are connected to a node which is already contained in the new cluster. Note that in the determination of the normalised degree for the unclustered nodes, the nodes that are already contained in the new cluster are still treated as unclustered nodes. The clause 'less than or equal to two' is absolutely essential to maintain the idea of a cluster not (directly) containing any branches. Doing this allows us to safely prune the hypergraph, i.e. cutting away irrelevant parts. Furthermore, any simple leaves ($\mathsf{Deg}(N, n) = 1$) connected to a node in the new cluster become part of the cluster as well. The definition of the Propagate function is now provided by:

$$\mathsf{Propagate} : \wp(\mathsf{Node}) \times (\mathbb{N} \rightarrowtail \wp(\mathsf{Node})) \times \mathbb{N} \to \wp(\mathsf{Node}) \times (\mathbb{N} \rightarrowtail \wp(\mathsf{Node}))$$

$$\mathsf{Propagate}(N, C, I) \triangleq \begin{cases} \mathsf{Propagate}(N - I, C \oplus_i I, i) & \text{if } I \neq \varnothing \\ \langle N, C \rangle & \text{otherwise} \end{cases}$$

where

$$I = \left\{ n \in N - C(i) \mid \exists_{n' \in C(i)} [\, \mathsf{NDeg}(N \cup C(i), n') \leq 2 \wedge n \leftrightsquigarrow n' \,] \right\}$$

$$\cup \left\{ n \in N - C(i) \mid \exists_{n' \in C(i)} [\, \mathsf{Deg}(N \cup C(i), n') = 1 \wedge n \leftrightsquigarrow n' \,] \right\}$$
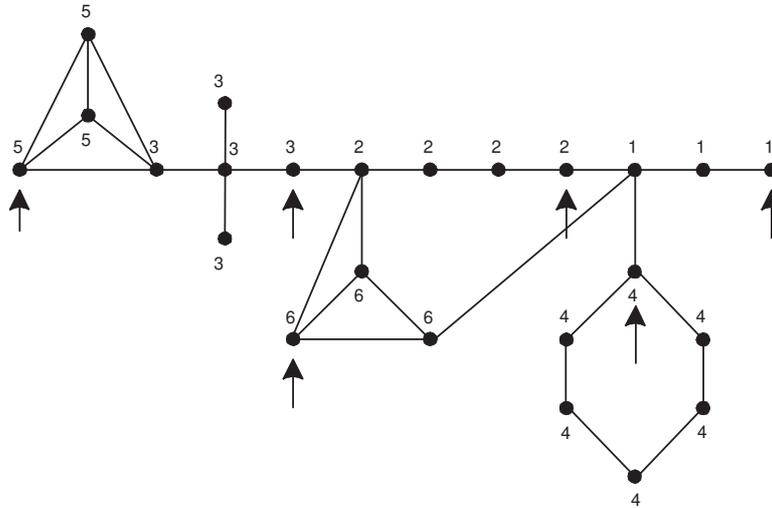
Figure 10: Example Clustering

As an example consider the graph depicted in figure 10. Each node has associated the number of the cluster it is part of. The arrows indicate the start node of each cluster.

Cluster 1: At the start of the algorithm four nodes have an NDeg of 1: the two right most nodes of cluster 1, and the top and bottom nodes of cluster 3. We arbitrarily chose for the first node of cluster 1 as our starting point.

The second node of cluster one is added because it has a neighbour (in cluster one) with an NDeg of 1. The third node (the most left one) of cluster 1 is then added to cluster one as it also has a neighbour (in cluster 1) with an NDeg of 1 (the middle node of cluster 1).

After adding the third node to cluster one, no more nodes can be added since the third node has an NDeg of 4. Now that cluster one cannot be extended any further a new cluster is formed.

Cluster 2: At this moment, again four nodes have an NDeg of 3: the two right most nodes of cluster 2, and the top and bottom nodes of cluster 3 (remember, the nodes from cluster 1 have now been 'removed' from the NDeg count).

Again we arbitrarily select the most right node of cluster 2 as a starting point. The three other nodes of cluster 2 are then added consecutively from right to left since they each have neighbours (in cluster 2) with an NDeg less than or equal to 2.

The last node added to cluster 2 has an NDeg of 4 and therefore no further nodes can be added.

Cluster 3: After the completion of cluster 2, once again four nodes with an NDeg of 3 remain. They are the four right most nodes of cluster 3. Note that the middle node of cluster 3 has an NDeg of 1 as well since it has only one neigbour with a degree higher than 1.

One arbitrary node is selected, and the other three nodes on cluster 3 are added consecutively.

Cluster 4, 5 and 6: All nodes that remain are the nodes of clusters 4, 5 and 6. All three clusters are simple cycles of three or six nodes. After selecting one node on each of the cycles, the remaining nodes on the cycles are added to the clusters.

For the clustering algorithm we can prove some useful properties. Firstly, the clustering algorithm leads to a partition of the nodes in the graph:

Lemma 5.1 For every graph G the function Cluster$(G.N)$ results in a partition of the nodes in graph G.

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Proof:

This follows immediately from the following observations:

1. The clustering algorithm only stops when all nodes are clustered (the $N = \varnothing$ clause in the definition of DoCluster).

2. The clustering algorithm removes every clustered nodes from the 'to do' set (the $N - \{n\}$, and $N - I$ clauses in the definition of DoCluster and Propagate respectively)

$\square$

The clusters do not contain nodes with a normalised degree that is higher than 2.

Lemma 5.2 If $C = \mathsf{Cluster}(G.N)$, then for all $i \in \mathsf{Dom}(C)$:

$$\forall_{n \in C(i)} \left[\mathsf{NDeg}(C(i), n) \leq 2 \right]$$

Proof:

Follows directly from the definition of I in the definition of Propagate. $\square$

The consequence of this lemma is, as stated before, that there are no real decision points (branches) within one cluster. A node may have more then one leaf neighbour, but will not contain more then two non leaf neighbours. The result of this is that we can safely treat the clusters of one (hyper) graph as nodes on the next hyper graph, and remove them if they are found irrelevant for the point to point query. The different levels of hypergraphs are built using the hyper cluster function which continues clustering until a graph results that does not contain any cycles:

$$\mathsf{HCluster} : \wp(\mathsf{Node}) \times \mathbb{N} \to \wp(\mathsf{Node}) \times \mathbb{N}$$

$$\mathsf{HCluster}(N, i) \triangleq \begin{cases} \langle N, i \rangle & \text{if } |E| = |N| - 1 \\ \mathsf{HCluster}(N', i+1) & \text{otherwise} \end{cases}$$

where

$$\begin{aligned} N' &= \mathsf{ran}(\mathsf{Cluster}(N)) \\ E &= \left\{ \{x, y\} \subseteq N \mid x \neq y \land x \leftrightsquigarrow y \right\} \end{aligned}$$

Note that $\mathsf{ran}(f)$ returns the range of function f. Not furthermore that a connected graph G is acyclic exactly when $|G.E| = |G.N| - 1$. The initial call of the hyper cluster function is $\mathsf{HCluster}(G.N, 1)$.
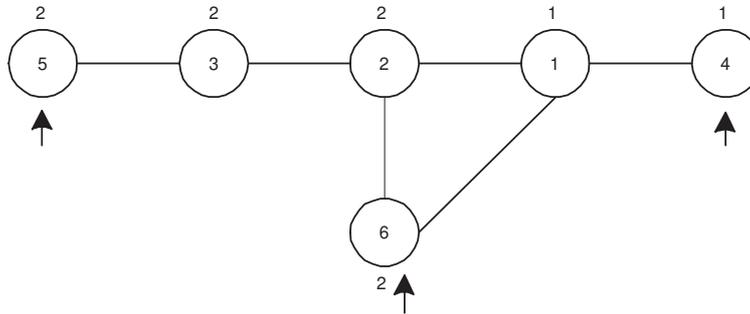

Figure 11: First Level Hypergraph

In figure 11 the hypergraph that can be associated to the clustering in figure 10 is depicted, together with a clustering of the hypergraph. Based on the clustering of this hypergraph, a second level hypergraph as depicted in figure 12 can be derived. As this graph is acyclic, the HCluster function terminates after the completion of this clustering.

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Figure 12: Second Level Hypergraph

5.2   The Complexity of Clustering

An important aspect of the clustering algorithm is the complexity of both the storage of the clusters as well as the algorithm itself. One call of the Cluster function is clearly linear in terms of the total number of nodes in the graph: $\Omega(|G.N|)$. The calculation of the complexity of the HCluster function, however, is a bit more complicated. The HCluster repeatedly tries to reduce the number of nodes in the (hyper) graph by calling the Cluster function. Trying to get a grips on the complexity of the HCluster function thus requires us to analyse the expected number of times that the Cluster function will be called, i.e. how many levels of hypergraphs we expect to have.
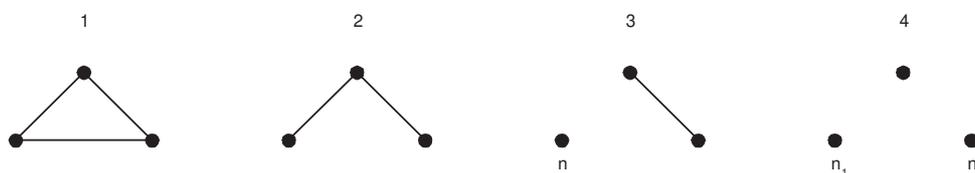

Figure 13: Alternative situations for remaining nodes

A first observation we make is that we are always dealing with a connected graph since a conceptual schema is a connected graph. We now prove that performing Cluster on a connected (hyper) graph with $n > 2$ nodes always leads to a connected hypergraph with maximally $n - 2$ nodes.

Lemma 5.3 $C = \text{Cluster}(N) \Rightarrow |\text{Dom}(C)| \leq |N| - 2$, i.e. applying Cluster leads to a reduction in the number of nodes of at least 2

Proof:

If $|N| = 3$, the Cluster(N) graph only contains 1 node, implying a reduction of exactly two nodes. This follows directly from the fact that in a graph with three nodes NDeg has a maximum value of 2.

Let $|N| > 3$. The Cluster function does not terminate until all nodes have been clustered. Now let us consider the last three nodes in N that are selected by Cluster to be clustered last. In figure 13, the four possible ways in which these three nodes can be connected are depicted. In the first two cases, the three nodes would be clustered together, thus leading to a reduction of at least two nodes. The two remaining cases require a closer examination:

1. In case three, the remaining three nodes obviously result in two clusters; leading to a reduction of only one node. However, we can prove that there must exist another cluster with at least two elements, thus implying that the total reduction size is still at least two.

   As the original graph is a connected one, node n must have been connected to some node(s) which are already clustered. Let M be the set of node(s) connected to node n that have been clustered the last. So the nodes in M are all part of the same cluster, and have the highest cluster number of n's neighbours. If M contains more then one node, this consequently means that there is at least one other cluster with more than two nodes, thus leading to a reduction of at least two nodes.

   If M contains only one node, say m, this implies that at the moment that the cluster containing node m was formed, node n was only connected to m. So node n is a leaf node of the graph at that moment with an NDeg of 1. This means that node n should have been clustered in the same cluster as m, which implies that M contains at least two nodes.

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2. In case four, the remaining three nodes lead to three separate clusters. However, we will prove that there are enough clusters with more then one element to ensure a reduction of 2 nodes.

Let M_1 be the set of node(s) connected to n_1 that were clustered last, and similarly M_2 the set of node(s) connected to n_2 that were clustered last. As the original graph was a connected graph, such nodes must exist.

If M_1 or M_2 contains only one node, then they should have been clustered already (same argument as above). So M_1 and M_2 both contain at least two nodes. This is not yet enough, since M_1 and M_2 may overlap. However, for $i \in \{1, 2\}$ we have the following:

If M_i has two elements, n_i must have an NDeg of at most 2. This means that one of the two nodes in M_j must have an NDeg of at most 2, since the clustering always starts with a node with the minimal NDeg. This in turn means that n_i is connected to a node with an NDeg that is less then or equal to two, and should therefore have been in the same cluster as M_i..

As a result, M_1 and M_2 contain at least three nodes. So even if they overlap the reduction of two nodes is still guaranteed.

<div align="right">□</div>

The connectedness of a hypergraph after applying Cluster to a connected graph follows directly from the way in which the edges are derived from the original graph. The above lemma allows us to identify the (execution) complexity of the HCluster algorithm. Since the number of nodes in the graph decreases by two in every clustering of the HCluster algorithm, the number of calls of HCluster to Cluster is limited to: $\lceil (|G.N|)/2 \rceil$. Therefore, the total complexity of the HCluster function is: $\Omega(|G.N|^2)$. However, since most conceptual schemes result in a sparse graph (not containing many edges), the results are likely to be better for most schemes.

Another important issue is the complexity of the memory used. Every clustering requires the storage of the nodes in the cluster. Let $n = |G.N|$, and $k = \lfloor n/2 \rfloor$, then we have the following worst case with respect to the number of nodes that need to be stored:

$$
\begin{aligned}
\Sigma_{j=0}^{k-1}(n - 2i) + 1 &= \Sigma_{i=1}^{k}(n + 2 - 2i) + 1 \\
&= nk + 2k + 1 - 2\Sigma_{i=1}^{k} i \\
&= nk + 2k + 1 - 2\frac{k(k+1)}{2} \\
&= nk + 2k + 1 - k(k+1) \\
&= nk - k^2 + k + 1 \\
&= \frac{n^2}{2} - \frac{n^2}{4} + \frac{n}{2} + 1 \\
&= \frac{n^2}{4} + \frac{n}{2} + 1
\end{aligned}
$$

As an example of a worst case graph, consider figure 14. This figure depicts the original graph, and the three associated hypergraphs after subsequent clustering steps. The original node contains 7 nodes, and it takes 3 steps to reduce it. The total number of nodes that needs to be stored is 16.

5.3 Reducing the Search Space

Using the hypergraphs, we can now reduce the size of the graph prior to searching the paths by means of the algorithm defined in section 4. In general, a (hyper)graph is reduced by:

$$
\mathsf{ReduceHG}_{f,t} : \wp(\mathsf{Node}) \to \wp(\mathsf{Node})
$$

$$
\mathsf{ReduceHG}_{f,t}(N) \triangleq \begin{cases} N & \text{if } N = N' \\ \mathsf{ReduceHG}_{f,t}(N) & \text{otherwise} \end{cases}
$$

Figure 14: Worst Case Clustering
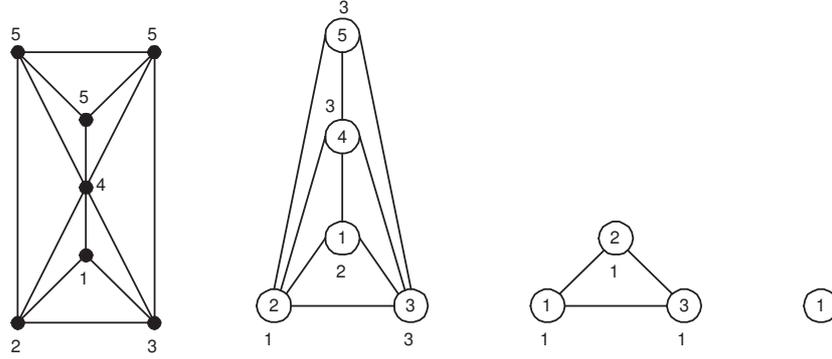
where
$$N' = \left\{ n \in N \mid \mathsf{SDeg}(N, n) > 1 \vee t \prec n \vee f \prec n \right\}$$

Note that, as mentioned before, the edges of the (hyper) graphs used in the above algorithm are derived from the original graph G. The *surrounding degree* (SDeg) of a node n in the (hyper) graph is the number of nodes in the original graph G that are reachable from n, and that are not contained in (or equal to) the current node n. These nodes are the surroundings of node n. The formal definition therefore is:

$$\mathsf{SDeg} : \wp(\mathsf{Node}) \times \mathsf{Node} \rightarrow \mathbb{N}$$
$$\mathsf{SDeg}(N, n) \triangleq \left| \left\{ y \in G.N \mid \exists_{m \in N - \{n\}} \left[n \leftrightsquigarrow m \wedge y \prec m \right] \right\} \right|$$
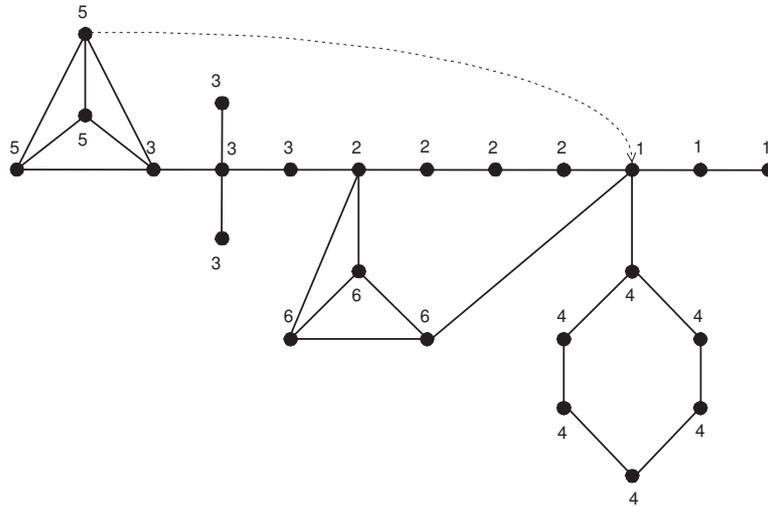

Figure 15: Example Point to Point Query

The reduction function $\mathsf{ReduceHG}_{f,t}$ simply removes all nodes that are neither the start nor the end of the point to point query and have a surrounding of only one node (e.g. subschema III in figure 8). The completely reduced graph is calculated by the following 'driver' function:

$$\mathsf{Reduce}_{f,t} : \wp(\mathsf{Node}) \times \mathbb{N} \rightarrow \wp(\mathsf{Edge}) \times \wp(\mathsf{Node})$$
$$\mathsf{Reduce}_{f,t}(N, n) \triangleq \begin{cases} \langle E', N' \rangle & \text{if } N = N' \\ \mathsf{Reduce}_{f,t}(\cup N', n-1) & \text{otherwise} \end{cases}$$
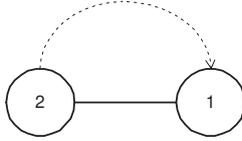
Figure 16: Point to Point Query on the Second Level Hyper Graph

where
$$N' \;=\; \mathsf{ReduceHG}_{f,t}(N) \text{ and } E' \;=\; \big\{\, \langle P, l \rangle \in E \;\big|\; P \subseteq N \,\big\}$$

If $\mathsf{HCluster}(G.N, 0) = \langle N', n \rangle$ for a certain graph G, then the search should be performed in the reduced graph: $\langle E'', N'' \rangle = \mathsf{Reduce}_{f,t}(N', n)$. The interesting question now is when $\mathsf{HCluster}(G.N, 0)$ should be calculated. Since this latter call does not depend on the point to point query specific source and destination, it could be calculated once after the completion of the conceptual schema (from which graph G is derived). Alternatively, one could calculate the hyper clustering each time a point to point query needs to be completed, which is likely to be very costly.
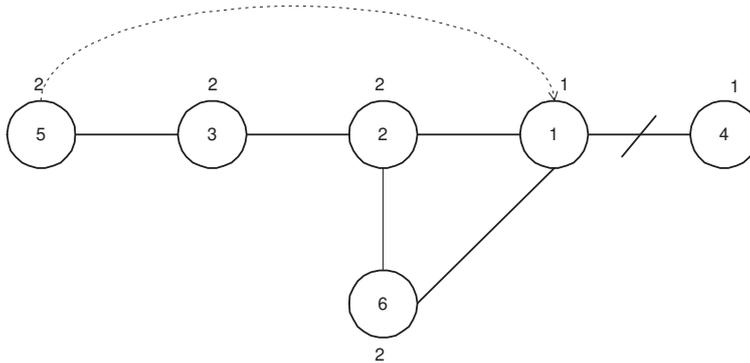

Figure 17: Point to Point Query on the First Level Hyper Graph

As an example reduction, consider the point to point query denoted in figure 15. The reduction algorithm starts with the reduction of the top level hypergraph as depicted in figure 16. Obviously, this graph cannot be reduced in any way. The next hypergraph that is considered by the reduction algorithm is shown in figure 17. Cluster 3 (containing the large cycle) is connected to only one other node and does not contain the start or end of the point to point query. Therefore, it can safely be removed from the graph. Finally, the original search graph can be reduced; the resulting graph is shown in figure 18. In this graph, the two leaf nodes from cluster 4 can be removed, as well as the two right nodes of cluster 1. Note that the nodes from cluster 3 have already been removed as the entire cluster was already removed in the previous step of the Reduce function.

6 Conclusions

In this article we introduced a novel way for computer supported query formulation called point to point queries. We provided a sample session with a provisional tool supporting point to point queries, and briefly discussed the relationship to query by navigation and query by construction. Together with these mechanisms a powerfull query formulation tool could be build. Furthermore, a search algorithm was introduced to search for the relevant paths between the specified points. Finally, an optimisation strategy for the search process was discussed based on a pre-compiled clustering of the conceptual schema graph.
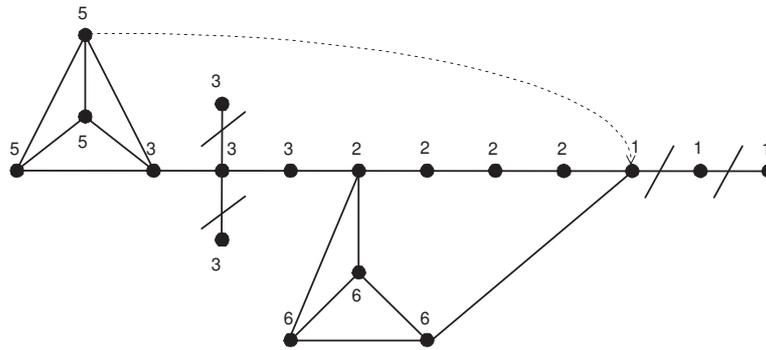
Figure 18: The Reduced Search Graph

As a next step, the path expressions should be further developed to suit our needs. Furthermore, elegant verbalisations of the path expressions should be catered for. asy

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